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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022
FIRST YEAR [BATCH 2021-24]

Date : 10/03/2022 **MATHEMATICS**
Time : 11am-1pm **Paper** : MTMA CC2 Full Marks : 50

Group A (2D Geometry and Vector Algebra)

Answer Question No. 1 and any two from Question No. 2 to Question No 4:-

1. Reduce the following equation to its canonical form and determine the type of the conic represented by it : $3x^2 + 10xy + 3y^2 - 2x - 14y - 13 = 0$. [5]

2. Find the equation of the circle which passes through the focus of the parabola $\frac{2a}{r} = 1 + \cos\theta$ and touches it at the point $\theta = \alpha$. [5]

3. Find the locus of the poles of the normal chords of the rectangular hyperbola $xy = c^2$. [5]

4. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, then find the area of the triangle formed by the bisectors of the angles between them and the axis of x. [5]

Answer any one from Question No. 5 to Question No. 6 :-

5. (a) If $\vec{\mathbf{a}} \cdot \vec{\mathbf{r}} = \alpha$, $\vec{\mathbf{b}} \cdot \vec{\mathbf{r}} = \beta$, $\vec{\mathbf{c}} \cdot \vec{\mathbf{r}} = \gamma$ and $\vec{\mathbf{d}} \cdot \vec{\mathbf{r}} = \delta$, then prove that [6]

$$\alpha[\vec{\mathbf{b}}\,\vec{\mathbf{c}}\,\vec{\mathbf{d}}] + \beta[\vec{\mathbf{c}}\,\vec{\mathbf{a}}\,\vec{\mathbf{d}}] + \gamma[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{d}}] = \delta[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}].$$

   (b) Solve for $\vec{\mathbf{r}}$, the vector equations $\vec{\mathbf{r}} \times \vec{\mathbf{a}} = \vec{\mathbf{b}}$ and $\vec{\mathbf{r}} \cdot \vec{\mathbf{a}} = p$, where $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are any two given non-zero vectors and p is a given scalar. Justify the solution geometrically. [3+1]

6. (a) A force $\vec{\mathbf{F}}$ of magnitude 10 units acts along the line $\frac{x-2}{5} = \frac{y-1}{4} = \frac{z-3}{3}$. Find the moment of the force $\vec{\mathbf{F}}$ about z axis. [5]

   (b) i. Find a set of vectors reciprocal to $\mathbf{2\hat{i}} + \mathbf{\hat{j}} - \mathbf{\hat{k}}$, $\mathbf{3\hat{i}} + \mathbf{2\hat{j}} + \mathbf{\hat{k}}$ and $\mathbf{\hat{i}} - \mathbf{\hat{j}} + \mathbf{2\hat{k}}$.

   ii. Express a vector $\vec{\mathbf{r}}$ as a linear combination of a vector $\vec{\mathbf{a}}$ and another vector perpendicular to $\vec{\mathbf{a}}$ and coplanar with $\vec{\mathbf{r}}$ and $\vec{\mathbf{a}}$. [4+1]

Group B (ODE 1)

Answer any one from Question No. 7 to 8 and any two from Question No. 9 to 11 :-

7. Solve the differential equation $x^3y^3(2y\,dx + x\,dy) - (5y\,dx + 7x\,dy) = 0$. [5]

8. Solve the differential equation [5]

$$\frac{d^5y}{dx^5} - \frac{dy}{dx} = e^x + \sin x - x.$$

9. (a) Reduce the following differential equation to Clairaut's form and hence solve it :

$$(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0,$$

   where $p = \frac{dy}{dx}$. [5]

(b) Solve by the method of undetermined coefficients : [5]

$$\frac{d^2y}{dt^2} - 9y = t + e^{2t} - \sin 2t.$$

10. (a) Solve by the method of variation of parameters : [6]

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = (1 + e^{-x})^{-1}$$

(b) Solve the equation [4]

$$\frac{dy}{dx} + \frac{x}{1 - x^2}y = x\sqrt{y}$$

11. (a) Solve : [6]

$$(1 + x)^2\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos\log(1 + x)$$

(b) Solve : $e^{p-y} = p^2 - 1$, where $p = \frac{dy}{dx}$. [4]

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